Observations on the analysis of interdependent creep processes

In many creep experiments conducted at high temperatures, two (or more) processes contribute towards the steady-state creep rate. When these processes are independent of each other, the observed creep rate, $\dot{\epsilon}$, for i different mechanisms is given by

$$\dot{\epsilon} = \sum_i \dot{\epsilon}_i \tag{1}$$

The situation becomes more complex when the processes are interdependent, such that the operation of one is a requirement for the operation of the others. An analysis for this situation was recently presented by Hay and Pascoe [1] in reporting an investigation of the creep behaviour of polycrystalline Fe_2O_3. Since their analysis has also been quoted in other work [2, 3], it appears appropriate to examine their approach.

According to Hay and Pascoe [1], geometrical integrity of the specimen requires that, for two interdependent processes, the strains are accumulated at equal rates, so that

$$\dot{\epsilon} = \dot{\epsilon}_1 = \dot{\epsilon}_2 \tag{2}$$

where $\dot{\epsilon}_1$ and $\dot{\epsilon}_2$ are the individual rates for the first and second process, respectively.

This condition is achieved by partitioning the applied stress, σ, into two effective stresses, σ_1 and σ_2, such that

$$\left.\begin{aligned}\dot{\epsilon}_1 &= k_1\sigma_1\\ \dot{\epsilon}_2 &= k_2\sigma_2^n\end{aligned}\right\} \tag{3}$$

where k_1, k_2, and n are constants.

Expressing the total work done as

$$\sigma\dot{\epsilon} = \sigma_1\dot{\epsilon}_1 + \sigma_2\dot{\epsilon}_2 \tag{4}$$

leads to

$$\sigma = \frac{\dot{\epsilon}}{k_1} + \left(\frac{\dot{\epsilon}}{k_2}\right)^{1/n} \tag{5}$$

Equation 5 was depicted by Hay and Pascoe [1] in their Figs. 1 and 2.

As first noted by Edington *et al.* [2] in a discussion of interdependent behaviour in superplasticity, the approach of Hay and Pascoe [1] is based on the unlikely assumption that both processes contribute equal strain rates under all experimental conditions. In general, this assumption is only valid in special cases which are not strictly representative of interdependent processes. For example, the approach is correct in a two-phase ($\alpha + \beta$) system, where both the α and β phases are forced to deform throughout their volumes at the same macroscopic strain rate: an example of this type of behaviour has been discussed by Kearns *et al.* [3]. In this case, the stress required to sustain a given strain rate in either phase depends, in addition, on the volume fractions of the two phases present.

However, in the more general case of a single-phase material, in which two (or more) processes operate in an interdependent manner, the observed value of $\dot{\epsilon}$ depends on the precise interaction between the processes. If one process can only occur *after* the other has taken place (*series-alternating, dependent* [4]), each process participates for a different increment of time through any period t. Thus,

$$t = t_1 + t_2 + t_3 + \ldots \tag{6}$$

Since integrity is preserved when each process contributes an identical strain, it follows that $\dot{\epsilon}$ is given by [4, 5]

$$\frac{1}{\dot{\epsilon}} = \sum_i \frac{1}{\dot{\epsilon}_i} \tag{7}$$

Alternatively, if one process can recommence operation *as soon as* the other begins (*series-sequential, dependent* [4]), the overall rate is then equal to the rate for the slowest process, so that

$$\dot{\epsilon} = \dot{\epsilon}_i|_{\dot{\epsilon}_i} \leqslant \dot{\epsilon}_1, \dot{\epsilon}_2, \dot{\epsilon}_3, \ldots \tag{8}$$

Equations 7 and 8 represent the two limiting situations for processes which are interdependent: in practice, it is anticipated that real materials will exhibit behaviour which is intermediate between these two extremes. As indicated schematically in Fig. 1, where $\dot{\epsilon}$ is logarithmically plotted against σ, the observed strain rate for two interdependent processes is in the range from $\dot{\epsilon}_1/2$ ($= \dot{\epsilon}_2/2$) to $\dot{\epsilon}_1$ ($= \dot{\epsilon}_2$) at the transition stress, σ_c, where the two mechanisms have equal strain rates. In general, the analysis due to Hay and Pascoe [1] gives a curve below that for series-alternating dependent processes in Fig. 1 (i.e., $\dot{\epsilon} < \dot{\epsilon}_1/2$ at $\sigma = \sigma_c$).

Full details of the present analysis are given elsewhere [4, 5]. It should be noted that the

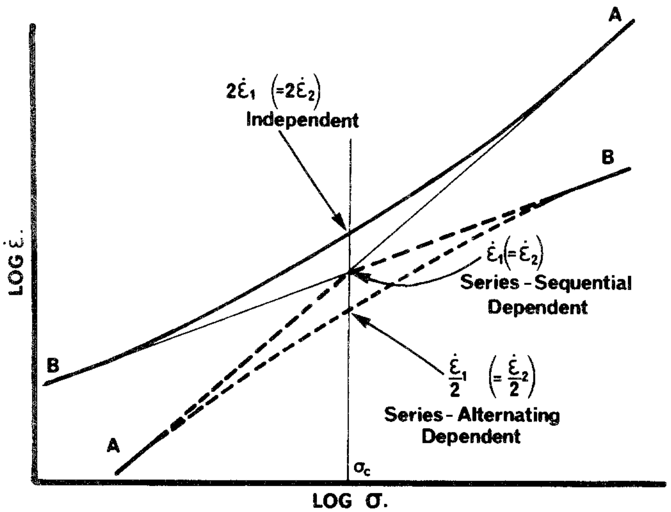

Figure 1 Schematic logarithmic plot of $\dot{\varepsilon}$ versus σ for creep mechanisms 1 and 2 represented by AA and BB, respectively, showing the resultant creep rate (full or broken lines) when the processes are independent or dependent.

approach represented by Equations 7 and 8, and depicted schematically in Fig. 1, is consistent with the analyses of Li [6] and Herriot *et al.* [7], with the exception that the latter authors specifically consider the possibility that one event of the first process may lead to X events of the second process. In this case, for two processes, Equation 6 is replaced by

$$t = t_1 + Xt_2 \qquad (9)$$

In summary, the analysis presented by Hay and Pascoe [1] is not generally valid for interdependent creep processes, except for special situations such as the deformation of two-phase systems. Alternative procedures are readily available [4–7].

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T. G. LANGDON*
R. C. GIFKINS
Department of Metallurgy,
University of Melbourne,
Parkville, Victoria 3052,
Australia

* Permanent address: Department of Materials Science, University of Southern California, Los Angeles, California 90007, USA.